Mathematics Assessment Framework

Grade 9

Bold= Eligible for CRT
Italics=Classroom Assessment Only
Normal=not included in 9th grade

Strand: ALGEBRA, FUNCTIONS, AND GRAPHS
Standard: Students will understand algebraic concepts and applications.

9-12 Benchmark: Represent and analyze mathematical situations and structures using algebraic symbols.

1. **Classify numbers and members of the following sets:**
   - **natural**
   - **whole**
   - **integers**
   - **rationals**
   - **irrationals**
2. **Simplify numerical expressions using the order of operations, including exponents.**
3. **Evaluate the numerical value of expressions of one or more variables that are:**
   - **polynomial**
   - **rational**
   - **radical**
4. **Simplify algebraic monomial expressions raised to a power (e.g., $[5xy^2\ ]^3$) and algebraic binomial (e.g., $[5x^2 + y]^2$) expressions raised to a power.**
5. Compare and order polynomial expressions by degree.
6. **Represent and analyze relationships using written and verbal expressions, tables, equations, and graphs, and describe the connections among those representations:**
   - **translate from verbal expression to algebraic formulae (e.g., "Set up the equations that represent the data in the following equation: John's father is 23 years older than John. John is 4 years older than his sister Jane. John's mother is 3 years younger than John's father. John's mother is 9 times as old as Jane. How old are John, Jane, John's mother, and John's father?")**
   - **given data in a table, construct a function that represents these data (linear only)**
   - **given a graph, construct a function that represents the graph (linear only)**
7. **Know, explain, and use equivalent representations for the same real number including:**
   - **integers**
   - **decimals**
   - **percents**
   - **ratios**
   - **scientific notation**
   - **numbers with integer exponents**
   - **inverses (reciprocal)**
   - **prime factoring**
8. **Simplify algebraic expressions using the distributive property.**
9. **Explain and use the concept of absolute value.**
10. **Know, explain, and use equivalent representations for algebraic expressions.**
11. **Simplify square roots and cube roots with monomial radicands that are perfect squares or**

perfect cubes (e.g., $9a^2x^4$).

12. Calculate powers and roots of real numbers, both rational and irrational.
13. **Solve:**
    - **formulas for specified variables**
    - radical equations involving one radical
14. **Factor polynomials, difference of squares and perfect square trinomials, and the sum and difference of cubes.**
15. Simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.
16. **Manipulate simple expressions with + and – exponents.**
17. **Use the four basic operations (+, -, x, ÷) with:**
    - **linear expressions**
    - **polynomial expressions**
    - rational expressions

9-12 Benchmark: Understand patterns, relations, functions, and graphs.

1. **Distinguish between the concept of a relation and a function.**
2. **Determine whether a relation defined by a graph, a set of ordered pairs, a table of values, an equation, or a rule is a function.**
3. *Describe the concept of a graph of a function.*
4. **Translate among tabular, symbolic, and graphical representations of functions.**
5. **Explain and use function notation.**
6. **Determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression.**
7. **Identify the independent and dependent variables from an application problem (e.g., height of a child).**
8. *Describe the concept of a graph of an equation.*
9. **Understand symmetry of graphs.**
10. Analyze and describe middle and end (asymptotic) behavior of linear, quadratic, and exponential functions, and sketch the graphs of functions.
11. Work with composition of functions (e.g., find f of g when $f(x) = 2x - 3$ and $g(x) = 3x - 2$), and find the domain, range, intercepts, zeros, and local maxima or minima of the final function.
12. Use the quadratic formula and factoring techniques to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.
13. Apply quadratic equations to physical phenomena (e.g., the motion of an object under the force of gravity).

9-12 Benchmark: Use mathematical models to represent and understand quantitative relationships.

1. **Model real-world phenomena using linear and quadratic equations and linear inequalities (e.g., apply algebraic techniques to solve rate problems, work problems, and percent mixture problems; solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest; apply quadratic equations to model throwing a baseball in the air).**
2. *Use a variety of computational methods (e.g., mental arithmetic, paper and pencil, technological tools).*
3. **Express the relationship between two variables using a table with a finite set of values and graph the relationship.**
4. **Express the relationship between two variables using an equation and a graph:**
    - **graph a linear equation and linear inequality in two variables**
    - **solve linear inequalities and equations in one variable**
    - **solve systems of linear equations in two variables and graph the solutions**
    - **use the graph of a system of equations in two variables to help determine the solution**
5. Solve applications involving systems of equations.
6. Evaluate numerical and algebraic absolute value expressions.
7. **Create a linear equation from a table of values containing co-linear data.**
8. **Determine the solution to a system of equations in two variables from a given graph.**
9. Generate an algebraic sentence to model real-life situations.
10. Write an equation of the line that passes through two given points.
11. Understand and use:
    - such operations as taking the inverse, finding the reciprocal, taking a root, and raising to a fractional power
    - the rules of exponents
12. **Verify that a point lies on a line, given an equation of the line**, and be able to derive linear equations by using the point-slope formula.

9-12 Benchmark: Analyze changes in various contexts.

1. **Analyze the effects of parameter changes on these functions:**
    - **linear (e.g., changes in slope or coefficients)**
    - quadratic (e.g., f[x-a] changes coefficients and constants)
    - exponential (e.g., changes caused by increasing x[x + c] or [a^x])
    - polynomial (e.g., changes caused by positive or negative values of a, or in a constant c)
2. **Solve routine two- and three-step problems relating to change using concepts such as:**
    - exponents
    - **factoring**
    - **ratio**
    - **proportion**
    - **average**
    - **percent**
3. **Calculate the percentage of increase and decrease of a quantity.**
4. Analyze the general shape of polynomial expressions and equations for different degree polynomials (e.g., positive and negative general shapes for third-, fourth-, and fifth-degree polynomials).
5. **Estimate the rate of change of a function or equation by finding the slope between two points on the graph.**
6. **Evaluate the estimated rate of change in the context of the problem.**
7. Know Pascal's triangle and use it to expand binomial expressions that are raised to positive integer powers.

Strand: GEOMETRY AND TRIGONOMETRY
Standard: Students will understand geometric concepts and applications.

9-12 Benchmark: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

1. **Interpret and draw two-dimensional objects and find the area and perimeter of basic figures (e.g., rectangles, circles, triangles, other polygons [e.g., rhombi, parallelograms, trapezoids]).**
2. **Find the area and perimeter of a geometric figure composed of a combination of two or more rectangles, triangles, and/or semicircles with just edges in common.**
3. **Find and use measures of sides and interior and exterior angles of triangles and polygons to classify figures (e.g., scalene, isosceles, and equilateral triangles; rectangles [square and non-square]; other convex polygons).**
4. **Interpret and draw three-dimensional objects and find the surface area and volume of basic figures (e.g., spheres, rectangular solids, prisms, polygonal cones), and calculate the surface areas and volumes of these figures as well as figures constructed from unions of rectangular solids and prisms with faces in common, given the formulas for these figures.**
5. Demonstrate an understanding of simple aspects of a logical argument:
    - identify the hypothesis and conclusion in logical deduction
    - use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion
6. Demonstrate an understanding of inductive and deductive reasoning, explain the difference between inductive and deductive reasoning, and identify and provide examples of each:
    - for inductive reasoning, demonstrate understanding that showing a statement is true for a finite number of examples does not show it is true for all cases unless the cases verified are all cases
    - for deductive reasoning, prove simple theorems
7. Write geometric proofs (including proofs by contradiction), including:
    - theorems involving the properties of parallel lines cut by a transversal line and the properties of quadrilaterals
    - theorems involving complementary, supplementary, and congruent angles
    - theorems involving congruence and similarity
    - the Pythagorean theorem (tangram proof)

9-12 Benchmark: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

1. **Demonstrate understanding of the construction of the coordinate plane, know the names of the origin, coordinate axes and four quadrants, draw and label them correctly, find the coordinates of an indicated point, and plot a point with given coordinates.**
2. **Determine the midpoint and distance between two points within a coordinate system and relate these ideas to geometric figures in the plane (e.g., find the center of a circle given two endpoints of a diameter of the circle).**
3. **Given two linear equations, determine whether the lines are parallel, perpendicular, or coincide.**
4. **Use basic geometric ideas (e.g., the Pythagorean Theorem, area, and perimeter of objects) in the context of the Euclidean Plane, calculate the perimeter of a rectangle with integer coordinates and sides parallel to the coordinate axes and with sides not parallel.**

9-12 Benchmark: Apply transformations and use symmetry to analyze mathematical situations.

1. **Describe the effect of rigid motions on figures in the coordinate plane and space that include rotations, translations, and reflections:**
    - **determine whether a given pair of figures on a coordinate plane represents the effect of a translation, reflection, rotation, and/or dilation**
    - **sketch the planar figure that is the result of a given transformation of this type**
2. **Deduce properties of figures using transformations that include translations, rotations, reflections, and dilations in a coordinate system:**
    - **identify congruency and similarity in terms of transformations**
    - **determine the effects of the above transformations on linear and area measurements of the original planar figure**

9-12 Benchmark: Use visualization, spatial reasoning, and geometric modeling to solve problems.

1. **Solve real-world problems using congruence and similarity relationships of triangles (e.g., find the height of a pole given the length of its shadow).**
2. **Solve problems involving complementary, supplementary, and congruent angles.**
3. **Solve problems involving the perimeter, circumference, area, volume, and surface area of common geometric figures (e.g., "Determine the surface area of a can of height h and radius r. How does the surface area change when the height is changed to 3h? How does the surface area change when the radius is changed to 3r? How does the surface area change when both h and r are doubled?").**
4. **Solve problems using the Pythagorean Theorem (e.g., "Given the length of a ladder and the distance of the base of the ladder from a wall, determine the distance up the wall to the top of the ladder").**
5. Understand and use elementary relationships of basic trigonometric functions defined by the angles of a right triangle (e.g., "What is the radius of a circle with an inscribed regular octagon with the length of each side being exactly 2 feet?").
6. Use trigonometric functions to solve for the length of the second leg of a right triangle given the angles and the length of the first leg. (e.g., "A surveyor determines that the angle subtended by a

two-foot stick at right angles to his transit is exactly one degree. What is the distance from the transit to the base of the measuring stick?").

7. Know and use angle and side relationships in problems with special right triangles (e.g., 30-, 45-, 60-, and 90-degree triangles).

Strand: DATA ANALYSIS AND PROBABILITY
Standard: Students will understand how to formulate questions, analyze data, and determine probabilities.

5-8 Benchmark: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

1. **Represent two numerical variables on a plot, describe how the data points are distributed, and identify relationships that exist between the two variables.**
2. **Generate, organize, and interpret real numbers in a variety of situations.**
3. **Organize, analyze, and display appropriate quantitative and qualitative data to address specific questions including:**
    - **frequency distributions**
    - **plots**
    - **histograms**
    - **bar, line, and pie graphs**
    - **diagram and pictorial displays**
    - **charts and tables**
4. **Select the appropriate measure of central tendency to describe a set of data for a particular problem situation.**
5. **Simulate an event selecting and using different models.**
6. **Develop an appropriate strategy using a variety of data from surveys, samplings, estimations, and inferences to address a specific problem.**

5-8 Benchmark: Select and use appropriate statistical methods to analyze data.

1. **Use changes in scales, intervals, or categories to help support a particular interpretation of data.**
2. *Generate*, **organize, and interpret real number and other data in a variety of situations.**
3. **Analyze data to make decisions and to develop convincing arguments from data displayed in a variety of formats that include:**
    - **plots**
    - **distributions**
    - **graphs**
    - **scatter plots**
    - **diagrams**
    - **pictorial displays**
    - **charts and tables**
    - **Venn diagrams**
4. **Interpret and analyze data from graphical representations and draw simple conclusions (e.g., line of best fit).**
5. **Evaluate and defend the reasonableness of conclusions drawn from data analysis.**
6. **Use appropriate central tendency and spread as a means for effective decision-making in analyzing data and outliers.**
7. **Identify simple graphic misrepresentations and distortions of sets of data (e.g., unequal interval sizes, omission of parts of axis range, scaling).**
8. *Use appropriate technology* **to display data as lists, tables, matrices, graphs, and plots and to analyze the relationships of variables in the data displayed.**

5-8 Benchmark: Develop and evaluate inferences and predictions that are based on data.

1. **Describe how changes in scale, intervals, or categories influence arguments for a particular interpretation of the data.**
2. **Describe how reader bias, measurement errors, and display distortion can affect the interpretation of data, predictions, and inferences based on data.**
3. *Conduct simple experiments and/or simulations, record results in charts, tables, or graphs,* **and use the results to draw conclusions and make predictions.**
4. **Compare expected results with experimental results and information used in predictions and inferences.**

5-8 Benchmark: Understand and apply basic concepts of probability.

1. **Calculate the odds of a desired outcome in a simple experiment.**
2. **Design and** *use* **an appropriate simulation to estimate the probability of a real-world event (e.g., disk toss, cube toss).**
3. **Explain the relationship between probability and odds and calculate the odds of a desired outcome in a simple experiment.**
4. **Use theoretical or experimental probability to make predictions about real-world events.**
5. **Use probability to generate convincing arguments, draw conclusions, and make decisions in a variety of situations.**

6. Understand that the probability of two unrelated events occurring is the sum of the two individual possibilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

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